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2009 J. Phys.: Condens. Matter 21 405406

(http://iopscience.iop.org/0953-8984/21/40/405406)

IOP PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **21** (2009) 499802 (1pp) doi:10.1088/0953-8984/21/49/499802

Corrigendum

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2009 *J. Phys.: Condens. Matter* **21** 405406

It has come to the attention of the authors that in the above article some errors occurred.

- There is an error in figure 2. The $+\infty$ and $-\infty$ should be replaced by $\frac{L_z}{2}$ and $-\frac{L_z}{2}$, respectively. L_z is the length of equilateral triangular quantum wire.

- Equation (54) should be replaced by

$$C_{lmk}^2 = \frac{32\pi^2}{\sqrt{3}L_z n^* \mu \left[4\left(l^2+m^2+lm\right)\pi^2+k^2A^2\right]}\left(\frac{n^*e}{1+\frac{8}{3}\pi n^*\alpha}\right)^2$$

$$= \frac{8\pi\omega_{\mathrm{LO}}^2}{\sqrt{3}L_z[4(l^2+m^2+lm)\pi^2+k^2A^2]}\left(\frac{1}{\varepsilon_\infty}-\frac{1}{\varepsilon_0}\right).$$

- Equation (62) should be replaced by

$$\Gamma_{lmk}^2 = \frac{8\pi\hbar e^2\omega_{\mathrm{LO}}}{\sqrt{3}L_z[4(l^2+m^2+lm)\pi^2+k^2A^2]}\left(\frac{1}{\varepsilon_\infty}-\frac{1}{\varepsilon_0}\right).$$

These errors do not affect the conclusions of the paper. The authors apologize for these errors and any possible inconvenience they have caused.

J. Phys.: Condens. Matter **21** (2009) 405406 (6pp) doi:10.1088/0953-8984/21/40/405406

Electron optical phonon interaction in equilateral triangular quantum dot and quantum wire

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Received 16 July 2009, in final form 27 August 2009
Published 14 September 2009
Online at stacks.iop.org/JPhysCM/21/405406

Abstract

The optical phonon modes and electron optical phonon interaction of the equilateral triangular quantum dot and quantum wire in vacuum are studied with the dielectric continuum model. The analytical expressions for the longitudinal optical phonon eigenfunctions are deduced. After having quantized the eigenmodes, we derive the Hamiltonian operators describing the longitudinal optical phonon modes and their interactions with electrons. The potential applications of these results are also discussed.

1. Introduction

In recent years, the challenge of synthetically controlling nanomaterial shape has been met with limited success. A variety of methods have been developed for making nanomaterials with a wide range of sizes, shapes and dielectric environments. The GaN and n-GaN/InGaN/p-GaN triangular nanowires [1, 2], PbSe, CdS, Ni, Ag, Au and NiS triangular nanoprisms (nanotriangles) [3–8] have been synthesized. Because of their special physical properties, they show successful and potential applications in a variety of fields such as nanowire lasers [9], optical biosensors [10] and photothermal agents [11]. The optoelectronic and physicochemical properties of nanomaterials are a strong function of particle size. Nanoparticle shape also contributes significantly to modulating their physical properties. Nanomaterials of different shapes have different crystallographic facets and have different fractions of surface atoms on their corners and edges, which makes it interesting to study the effect of shape on their physical properties. It is well known that the electron–phonon interaction is an important factor influencing the physical properties of the polar crystals such as the binding energy of impurities, carrier transportation, linear and nonlinear optical properties, especially in low-dimensional materials. Hence, a proper description of polar optical phonon modes and the electron–phonon interaction Hamiltonian for quantum systems of complex shapes is necessary.

Many authors have made their contributions in studying phonon modes and electron–phonon interaction in various low-dimensional quantum systems. The electron–phonon interactions in a dielectric confined system was first studied by Fuchs [12], Lucas [13] and Licari [14] within the dielectric continuum model. Wendler [15, 16] developed the framework of the theory of optical phonons and electron–phonon interaction for the spatially confined systems. Mori and Ando [17] have determined the phonon modes in single and double heterostructures. Stroscio [18] has deduced the longitudinal optical (LO) and surface optical (SO) phonon modes in a rectangular quantum wire. Xie *et al* [19, 20] have studied the interface optical (IO) and SO phonon modes in a cylindrical quantum well wire with a finite confining potential and an infinite potential. Klein [21] and Roca [22] have derived the polar optical phonon modes in spherical quantum dots. Cruz [23] has obtained the IO phonon mode in GaAs/Al$_x$Ga$_{1-x}$As quantum spheres. Li and Chen [24] have deduced the LO, top surface optical (TSO) and side surface optical (SSO) phonon modes in a cylindrical quantum dot. Zhang [25] has studied the phonon modes in a quantum dot quantum well. Wu and Xie [26] have worked out the LO, TSO and SSO phonon modes in a quantum annulus. However, in theory, the exact formulations for the phonon modes and the electron–phonon interaction in quantum systems of complex

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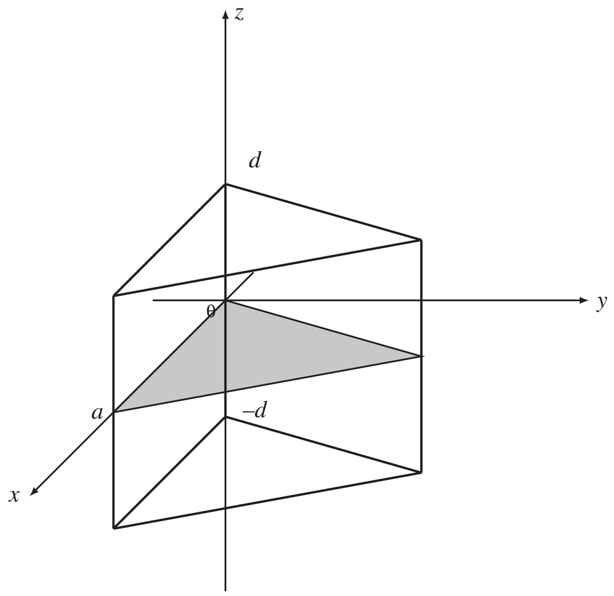

Figure 1. The geometry of ETQD.

shapes such as equilateral triangular quantum dots (ETQD) and equilateral triangular quantum wires (ETQW) are still absent.

In this paper, we consider the ETQD and ETQW in vacuum with the dielectric continuum model. This paper is organized as follows: in section 2, the confined LO phonon modes and the corresponding Fröhlich electron–phonon interaction Hamiltonian of the ETQD are deduced. In section 3, by the theoretical scheme of the ETQD, we derive the confined LO phonon modes and the corresponding Fröhlich electron–phonon interaction Hamiltonian of the ETQW. In section 4, the potential applications of these results are discussed.

2. The confined LO phonon modes of ETQD

Firstly, we consider an ETQD of polar semiconductor placed in vacuum. The geometry of the ETQD is shown in figure 1. The height is $2d$. The side with equilateral triangular cross section is a. The dielectric constant is assumed to be isotropic.

Under the dielectric continuum approximation, we start with the electrostatic equations

$$\mathbf{D} = \varepsilon\mathbf{E} = \mathbf{E} + 4\pi\mathbf{P}, \tag{1}$$

$$\mathbf{E} = -\nabla\phi(\mathbf{r}), \tag{2}$$

$$\nabla\cdot\mathbf{D} = 4\pi\rho_0(\mathbf{r}), \tag{3}$$

where $\mathbf{D}$, $\mathbf{E}$, $\mathbf{P}$ and ϕ are the electric displacement, electric field strength, electric polarization density and electric potential, respectively. ρ_0 is the free charge density and ε is the dielectric constant. For free oscillation, the charge density $\rho_0(\mathbf{r}) = 0$, so we get the following equation:

$$\varepsilon\nabla^2\phi(\mathbf{r}) = 0. \tag{4}$$

There are two possible solutions for equation (4), one of which is

$$\varepsilon(\omega) = 0, \tag{5}$$

and the other is

$$\nabla^2\phi(\mathbf{r}) = 0. \tag{6}$$

In this paper, we only focus on the first solution. Since in a polar crystal

$$\varepsilon(\omega) = \varepsilon_\infty + \frac{\varepsilon_0 - \varepsilon_\infty}{1 - \omega^2/\omega_{\mathrm{TO}}^2}, \tag{7}$$

where ε_0 and ε_∞ are the static and high-frequency dielectric constants and ω_{TO} is the frequency of the transverse optical phonon. $\varepsilon(\omega) = 0$ would give

$$\omega^2 = \omega_{\mathrm{TO}}^2\frac{\varepsilon_0}{\varepsilon_\infty} = \omega_{\mathrm{LO}}^2, \tag{8}$$

Equation (8) is just the Lyddane–Sachs–Teller (LST) relation, which describes the bulk LO modes of frequency $\omega = \omega_{\mathrm{LO}}$.

2.1. The eigenfunctions of the confined LO phonons

In the ETQD, the electric potential $\phi(\mathbf{r})$ in equation (4) is an arbitrary function of x, y and z. Owing to the electrostatics boundary conditions that the tangential component of $\mathbf{E}$ and the normal component of $\mathbf{D}$ are continuous at the boundary and equation (5), the electric potential $\phi(\mathbf{r})$ should be zero at the boundary and in the region outside. According to the geometry of the quantum dot, the electric potential $\phi(\mathbf{r})$ can be taken as

$$\phi(\mathbf{r}) = \Psi(x, y)f(z), \tag{9}$$

$$f(z) = \begin{cases} C\cos(k_z z) + D\sin(k_z z) & \text{for } -d \leqslant z \leqslant d, \\ 0 & \text{otherwise}, \end{cases} \tag{10}$$

where the k_z is the phonon wavevector in the z direction. C, D and k_z can be determined by the boundary conditions. Since the place $z = 0$ possesses reflectional symmetry, the electrostatic potential should be either symmetric or antisymmetric about this place. The boundary conditions match in two ways: either $C = 0$ or $D = 0$. Hence, we have two solutions for $f(z)$:

$$f^S(z) = C\cos\left(\frac{k\pi}{2d}z\right), \qquad k = 1, 3, 5, \ldots, \tag{11}$$

and

$$f^A(z) = D\sin\left(\frac{k\pi}{2d}z\right), \qquad k = 2, 4, 6, \ldots. \tag{12}$$

However, the equilateral triangular cross section function $\Psi(x, y)$ is not solvable by the separation of variables. In fact, the cross-section function problem is similar to the quantum mechanical problem of a particle in an equilateral triangle infinite well (or billiard). So, the eigenfunctions of the Schrödinger equation can be chosen as the cross-section function, which can be found in these articles [27–35]. To make the paper self-contained, we reproduce the results of Li and Blinder [27]. A particle with the mass m in the equilateral triangle is described by the Schrödinger equation:

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\Psi(x, y) = E\Psi(x, y). \tag{13}$$

Firstly, it is convenient to introduce the three auxiliary variables:

$$u = \frac{2\pi}{A} y, \qquad v = \frac{2\pi}{A}\left(-\frac{y}{2} + \frac{\sqrt{3}}{2}x\right),$$
$$w = \frac{2\pi}{A}\left(-\frac{y}{2} - \frac{\sqrt{3}}{2}x\right) + 2\pi, \qquad u + v + w = 2\pi, \tag{14}$$

where $A = \sqrt{3}a/2$ is the altitude of the equilateral triangular cross section.

The C_{3v} group of the triangle is isomorphous with the group S_3 representing the permutations among the variables u, v and w. The C_{3v} group has three inequivalent irreducible representations A_1, A_2 (one-dimensional) and E (two-dimensional). By way of the projection operators of the irreducible representations, we can construct C_{3v}-symmetry-adapted functions. The A_1, A_2 and E projection operators are as follows:

$$P(A_1) = E + C_2 + C_2^2 + \delta_1 + \delta_2 + \delta_3, \tag{15}$$
$$P(A_2) = E + C_2 + C_2^2 - \delta_1 - \delta_2 - \delta_3, \tag{16}$$
$$P(E) = E + \eta C_2 + \eta^* C_2^2 - \delta_1 \pounds - \eta\delta_2 \pounds - \eta^*\delta_3 \pounds, \tag{17}$$
$$\eta = e^{(\pm\frac{2\pi i}{3})}, \tag{18}$$

where E, C_2, C_2^2, δ_1, δ_2 and δ_3 are the six symmetry operations of the C_{3v} group. $\pounds$ stands for the operation of complex conjugation. Boundary conditions aside, the Schrödinger equation (13) admits a harmonic function solution of the form $f(lu - mv)$, in which l and m are constants. The wavefunction vanishes, when $u = 0$, $v = 0$ and $w = 0$. To find the boundary conditions, f must represent the functions sin, cos and exp($\pm$i), respectively, for A_1, A_2 and E eigenfunctions. The explicit forms are as follows:

$$\Psi_{lm}^{A_1} = \sin(lu - mv) + \sin(lv - mw) + \sin(lw - mu)$$
$$+ \sin(lu - mw) + \sin(lw - mv) + \sin(lv - mu), \tag{19}$$

where $m = 0, 1, 2, \ldots$, $l = m + 1, m + 2, \ldots$, and

$$\Psi_{lm}^{A_2} = \cos(lu - mv) + \cos(lv - mw) + \cos(lw - mu)$$
$$- \cos(lu - mw) - \cos(lw - mv) - \cos(lv - mu), \tag{20}$$

where $m = 1, 2, 3, \ldots$, $l = m + 1, m + 2, \ldots$, and

$$\Psi_{lm}^{E} = e^{i(lu-mv)} + \eta e^{i(lu-mw)} + \eta^* e^{i(lw-mu)}$$
$$- e^{-i(lv-mu)} - \eta e^{-i(lw-mv)} - \eta^* e^{-i(lu-mw)}, \tag{21}$$

where for $\eta = \exp(2\pi i/3)$, $m = 1/3, 4/3, 7/3, \ldots$, $l = m+1$, $m + 2, \ldots$, while $\eta = \exp(-2\pi i/3)$, $m = 2/3, 5/3, 8/3, \ldots$, $l = m + 1, m + 2, \ldots$.

Using $w = 2\pi - u - v$, and the l, m values, the A_1, A_2 and E eigenfunctions can be reduced to more compact trigonometric forms. We can divide these eigenfunctions into the symmetric part $\Psi_{lm}^{+}(x, y)$ and the antisymmetric part $\Psi_{lm}^{-}(x, y)$:

$$\Psi_{lm}^{+}(x, y) = \frac{1}{\sqrt{\delta_{0m} + 1}}\left\{\cos\left(\frac{\sqrt{3}\pi m}{A}x\right)\sin\left[\frac{(2l+m)\pi}{A}y\right]\right.$$
$$- \cos\left(\frac{\sqrt{3}\pi l}{A}x\right)\sin\left[\frac{(2m+l)\pi}{A}y\right]$$
$$\left.- \cos\left[\frac{\sqrt{3}\pi(l+m)}{A}x\right]\sin\left[\frac{(l-m)\pi}{A}y\right]\right\}, \tag{22}$$

where $m = 0, 1/3, 2/3, 1, 4/3, 5/3, \ldots$, $l = m + 1, m + 2, \ldots$, δ_{0m} is the Kronecker delta function and

$$\Psi_{lm}^{-}(x, y) = \sin\left(\frac{\sqrt{3}\pi m}{A}x\right)\sin\left[\frac{(2l+m)\pi}{A}y\right]$$
$$- \sin\left(\frac{\sqrt{3}\pi l}{A}x\right)\sin\left[\frac{(2m+l)\pi}{A}y\right]$$
$$+ \sin\left[\frac{\sqrt{3}\pi(l+m)}{A}x\right]\sin\left[\frac{(l-m)\pi}{A}y\right], \tag{23}$$

where $m = 1/3, 2/3, 1, 4/3, 5/3, \ldots$, $l = m + 1, m + 2, \ldots$. The functions $\Psi_{lm}^{+}(x, y)$ and $\Psi_{lm}^{-}(x, y)$ form a complete orthogonal set on the cross section [32–34].

So, the eigenfunctions of the confined LO phonons can be chosen as

$$\phi_{lmk}^{S+} = \begin{cases} C_{lmk}\Psi_{lm}^{+}(x, y) f^S(z) & \text{in ETQD,} \\ 0 & \text{otherwise,} \end{cases} \tag{24}$$

$$\phi_{lmk}^{S-} = \begin{cases} C_{lmk}\Psi_{lm}^{-}(x, y) f^S(z) & \text{in ETQD,} \\ 0 & \text{otherwise,} \end{cases} \tag{25}$$

for $k = 1, 3, 5, \ldots$, and

$$\phi_{lmk}^{A+} = \begin{cases} C_{lmk}\Psi_{lm}^{+}(x, y) f^A(z) & \text{in ETQD,} \\ 0 & \text{otherwise,} \end{cases} \tag{26}$$

$$\phi_{lmk}^{A-} = \begin{cases} C_{lmk}\Psi_{lm}^{-}(x, y) f^A(z) & \text{in ETQD,} \\ 0 & \text{otherwise,} \end{cases} \tag{27}$$

for $k = 2, 4, 6, \ldots$.

The polarization vectors for the confined LO mode are calculated by considering equations (1) and (2) and the condition $\varepsilon = 0$. We get

$$\mathbf{P}_{lmk}^{\sigma} = \frac{1}{4\pi}\nabla\phi_{lmk}^{\sigma}, \qquad \sigma = S+, S-, A+, A-. \tag{28}$$

2.2. The phonon modes and electron–phonon interaction Hamiltonians

To find the expression for the Hamiltonian of the free-phonon field, we start with the dynamic equations of motion of the crystal lattice [14]:

$$\mu\ddot{\mathbf{u}} = \mu\omega_0^2\mathbf{u} + e\mathbf{E}_{\text{loc}}, \tag{29}$$

$$\mathbf{P} = n^* e\mathbf{u} + n^*\alpha\mathbf{E}_{\text{loc}}, \tag{30}$$

where $\mu = m_+ m_- / (m_+ + m_-)$ is the reduced mass of the ion pair and $\mathbf{u} = \mathbf{u}_+ - \mathbf{u}_-$ is the relative displacement of the positive and negative ions, ω_0 is the frequency associated with the short-range force between ions, n^* is the number of ion pairs per unit volume, α is the electronic polarizability per ion pair and $\mathbf{E}_{\text{loc}}$ is the local field at the position of the ions. $\mathbf{P}$ is the polarization field produced by the oscillating ions. The Hamiltonian of the free vibration is given by

$$H_{\text{ph}} = \tfrac{1}{2}\int [n^*\mu\dot{\mathbf{u}}\cdot\dot{\mathbf{u}} + n^*\mu\omega_0^2\mathbf{u}\cdot\mathbf{u} - n^* e\mathbf{u}\cdot\mathbf{E}_{\text{loc}}]\,d^3r. \tag{31}$$

Using the well-known Lorentz relation $\mathbf{E}_{\mathrm{loc}} = \mathbf{E} + 4\pi\mathbf{P}/3$ and the relation $\mathbf{E} = -4\pi\mathbf{P}$, we have

$$\mathbf{E}_{\mathrm{loc}} = -\tfrac{8}{3}\pi\mathbf{P}, \tag{32}$$

$$\mathbf{u} = \frac{1+\frac{8}{3}\pi n^*\alpha}{n^*e}\mathbf{P}. \tag{33}$$

Substituting equations (32) and (33) into (29), we can derive

$$\ddot{\mathbf{u}} + \omega_{\mathrm{LO}}^2\mathbf{u} = 0, \tag{34}$$

where

$$\omega_{\mathrm{LO}}^2 = \omega_0^2 + \frac{8}{3}\frac{\pi n^* e^2/\mu}{1+\frac{8}{3}\pi n^*\alpha}. \tag{35}$$

Hence, the confined LO phonon Hamiltonian from equation (31) can be written as

$$H_{\mathrm{LO}} = \frac{1}{2}\int\left[n^*\mu\left(\frac{1+\frac{8}{3}\pi n^*\alpha}{n^*e}\right)^2(\dot{\mathbf{P}}^*\cdot\dot{\mathbf{P}} + \omega_{\mathrm{LO}}^2\mathbf{P}^*\cdot\mathbf{P})\right]\mathrm{d}^3r. \tag{36}$$

The LO polarization vectors from equation (28) form an orthonormal and complete set:

$$\begin{aligned}&\int 2n^*\mu\left(\frac{1+\frac{8}{3}\pi n^*\alpha}{n^*e}\right)^2\mathbf{P}^{\sigma'*}_{l'm'k'}\cdot\mathbf{P}^{\sigma}_{lmk}\,\mathrm{d}^3r\\ &= \frac{n^*\mu}{8\pi^2}\left(\frac{1+\frac{8}{3}\pi n^*\alpha}{n^*e}\right)^2\int\nabla\phi^{\sigma'*}_{l'm'k'}\cdot\nabla\phi^{\sigma}_{lmk}\,\mathrm{d}^3r\\ &= \frac{-n^*\mu}{8\pi^2}\left(\frac{1+\frac{8}{3}\pi n^*\alpha}{n^*e}\right)^2\int\phi^{\sigma'*}_{l'm'k'}\nabla^2\phi^{\sigma}_{lmk}\,\mathrm{d}^3r.\end{aligned} \tag{37}$$

in which we use the Green's first identity:

$$\int_V\nabla\phi\cdot\nabla\varphi\,\mathrm{d}^3r = -\int_V\phi\nabla^2\varphi\,\mathrm{d}^3r + \int_S\phi\frac{\partial\varphi}{\partial n}\,\mathrm{d}S, \tag{38}$$

In our case, $\phi \equiv 0$ (on the boundary), so that the second term equals zero. We get

$$\int 2n^*\mu\left(\frac{1+\frac{8}{3}\pi n^*\alpha}{n^*e}\right)^2\mathbf{P}^{\sigma'*}_{l'm'k'}\cdot\mathbf{P}^{\sigma}_{lmk}\,\mathrm{d}^3r = \delta_{\sigma\sigma'}\delta_{ll'}\delta_{mm'}\delta_{kk'}, \tag{39}$$

from equation (39) C_{lmk} can be determined

$$\begin{aligned}C_{lmk}^2 &= \frac{128d}{\sqrt{3}n^*\mu\left[16(l^2+m^2+lm)d^2+k^2A^2\right]}\left(\frac{n^*e}{1+\frac{8}{3}\pi n^*\alpha}\right)^2\\ &= \frac{32d\omega_{\mathrm{LO}}^2}{\sqrt{3}\pi[16(l^2+m^2+lm)d^2+k^2A^2]}\left(\frac{1}{\varepsilon_\infty}-\frac{1}{\varepsilon_0}\right),\end{aligned} \tag{40}$$

in which we use the ω_{TO} relation [14]:

$$\omega_{\mathrm{TO}}^2 = \omega_0^2 - \frac{4}{3}\frac{\pi n^*e^2/\mu}{1-\frac{4}{3}\pi n^*\alpha}, \tag{41}$$

and the Clausius–Mossotti relation:

$$\varepsilon_\infty = 1 + \frac{4\pi n^*}{1-\frac{4}{3}\pi n^*\alpha}. \tag{42}$$

We can express the polarization field $\mathbf{P}$ in terms of the complete set of orthonormal polarization modes $\mathbf{P}^{\sigma}_{lmk}$:

$$\mathbf{P} = \sum_{lmk}\left(\frac{\hbar}{\omega_{\mathrm{LO}}}\right)^{\frac{1}{2}}(a^\dagger_{lmk}+a_{lmk})\mathbf{P}_{lmk}, \tag{43}$$

$$\dot{\mathbf{P}} = -\mathrm{i}\sum_{lmk}(\hbar\omega_{\mathrm{LO}})^{\frac{1}{2}}(a^\dagger_{lmk}-a_{lmk})\mathbf{P}_{lmk}. \tag{44}$$

$\mathbf{P}$ and $\dot{\mathbf{P}}$ are now quantum field operators. $a^\dagger_{lmk}$ and a_{lmk} are creation and annihilation operators for the LO phonon of the (l, m, k)th mode. They satisfy the commutative rules for the boson commutation relation:

$$[a_{lmk}, a^\dagger_{l'm'k'}] = \delta_{ll'}\delta_{mm'}\delta_{kk'}, \tag{45}$$

$$[a_{lmk}, a_{l'm'k'}] = [a^\dagger_{lmk}, a^\dagger_{l'm'k'}] = 0. \tag{46}$$

Hence, from equations (43)–(46), the Hamiltonian operator for confined LO phonon becomes

$$H_{\mathrm{LO}} = \sum_{lmk}\hbar\omega_{\mathrm{LO}}\left(a^\dagger_{lmk}a_{lmk}+\tfrac{1}{2}\right). \tag{47}$$

The electric potential can be expanded as

$$\phi_{\mathrm{LO}} = \sum_{lmk}\left(\frac{\hbar}{\omega_{\mathrm{LO}}}\right)^{\frac{1}{2}}(a_{lmk}\phi^{\sigma}_{lmk}+\mathrm{h.c.}), \tag{48}$$

where h.c. means the Hermitian conjugate.

The Fröhlich Hamiltonian between the electron and LO phonon can then be written as

$$H_{\mathrm{e-LO}} = -e\phi_{\mathrm{LO}} = -\sum\nolimits_{lmk}(\Gamma_{lmk}a^\dagger_{lmk}\phi^{\sigma}_{lmk}+\mathrm{h.c.}), \tag{49}$$

where

$$\Gamma^2_{lmk} = \frac{32\hbar e^2\,\mathrm{d}\omega_{\mathrm{LO}}}{\sqrt{3}\pi[16(l^2+m^2+lm)d^2+k^2A^2]}\left(\frac{1}{\varepsilon_\infty}-\frac{1}{\varepsilon_0}\right). \tag{50}$$

3. The confined LO phonon modes of ETQW

For the ETQW, a parallel development is possible for the determination of the longitudinal optical phonon mode. Since the ETQW (see figure 2) is translationally invariant in the z direction, the eigenfunctions of the confined LO phonons can be chosen as

$$\phi^{\pm}_{lmk} = \begin{cases}C_{lmk}\Psi^{\pm}_{lm}(x,y)\mathrm{e}^{\mathrm{i}kz} & \text{in ETQW,}\\ 0 & \text{otherwise,}\end{cases} \tag{51}$$

where the k is the phonon wavevector in the z direction. The polarization vectors for the confined LO mode are

$$\mathbf{P}^{\pm}_{lmk} = \frac{1}{4\pi}\nabla\phi^{\pm}_{lmk}. \tag{52}$$

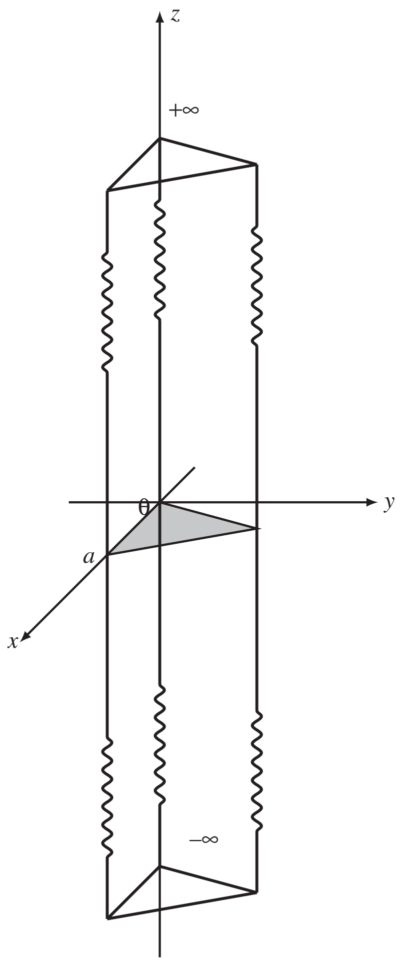

Figure 2. The geometry of ETQW.

The LO polarization vectors form an orthonormal and complete set:

$$\int 2n^*\mu\left(\frac{1+\frac{8}{3}\pi n^*\alpha}{n^*e}\right)^2 \mathbf{P}^{i*}_{l'm'k'}\cdot\mathbf{P}^{j}_{lmk}\,\mathrm{d}^3r = \delta_{ij}\delta_{ll'}\delta_{mm'}\delta_{kk'}, \tag{53}$$

$$C^2_{lmk} = \frac{16\pi}{\sqrt{3}n^*\mu\left[4\left(l^2+m^2+lm\right)\pi^2+k^2A^2\right]} \times\left(\frac{n^*e}{1+\frac{8}{3}\pi n^*\alpha}\right)^2 = \frac{4\omega^2_{\mathrm{LO}}}{\sqrt{3}[4(l^2+m^2+lm)\pi^2+k^2A^2]}\left(\frac{1}{\varepsilon_\infty}-\frac{1}{\varepsilon_0}\right). \tag{54}$$

The polarization field $\mathbf{P}$ can be expressed in terms of the complete set of orthonormal polarization modes $\mathbf{P}^{\pm}_{lmk}$:

$$\mathbf{P} = \sum_{lmk}\left(\frac{\hbar}{\omega_{\mathrm{LO}}}\right)^{\frac{1}{2}}(a^{\dagger}_{lmk}+a_{lmk})\mathbf{P}_{lmk}, \tag{55}$$

$$\dot{\mathbf{P}} = -\mathrm{i}\sum_{lmk}(\hbar\omega_{\mathrm{LO}})^{\frac{1}{2}}(a^{\dagger}_{lmk}-a_{lmk})\mathbf{P}_{lmk}. \tag{56}$$

$\mathbf{P}$ and $\dot{\mathbf{P}}$ are now quantum field operators. $a^{\dagger}_{lmk}$ and a_{lmk} are creation and annihilation operators for the LO phonon of the (l, m, k)th mode. They satisfy the commutative rules for the boson commutation relation:

$$[a_{lmk}, a^{\dagger}_{l'm'k'}] = \delta_{ll'}\delta_{mm'}\delta_{kk'}, \tag{57}$$

$$[a_{lmk}, a_{l'm'k'}] = [a^{\dagger}_{lmk}, a^{\dagger}_{l'm'k'}] = 0. \tag{58}$$

The Hamiltonian operator for confined LO phonon becomes

$$H_{\mathrm{LO}} = \sum_{lmk}\hbar\omega_{\mathrm{LO}}(a^{\dagger}_{lmk}a_{lmk}+\tfrac{1}{2}). \tag{59}$$

The electric potential can be expanded as

$$\phi_{\mathrm{LO}} = \sum_{lmk}\left(\frac{\hbar}{\omega_{\mathrm{LO}}}\right)^{\frac{1}{2}}(a_{lmk}\phi^{\pm}_{lmk}+\mathrm{h.c.}). \tag{60}$$

The Fröhlich Hamiltonian between the electron and LO phonon can then be written as

$$H_{\mathrm{e-LO}} = -e\phi_{\mathrm{LO}} = -\sum_{lmk}(\Gamma_{lmk}a^{\dagger}_{lmk}\phi^{\pm}_{lmk}+\mathrm{h.c.}), \tag{61}$$

where

$$\Gamma^2_{lmk} = \frac{4\hbar e^2\omega_{\mathrm{LO}}}{\sqrt{3}[4(l^2+m^2+lm)\pi^2+k^2A^2]}\left(\frac{1}{\varepsilon_\infty}-\frac{1}{\varepsilon_0}\right). \tag{62}$$

4. Summary and discussion

In this paper, by using the technique of group theory, we derived the analytical expressions for the confined LO phonon modes, the Hamiltonian operators for confined LO phonon and the Fröhlich Hamiltonian between the electron and LO phonon in the ETQD and ETQW. This is a starting point for the study of the polaron effect and the phonon-assisted physical processes in these systems. For example, with the Fermi golden rule, we can apply the expressions to determine the electron–LO phonon scattering rates. And, what is more, similar mathematic techniques can be used to study the phonon modes in the quantum dot and quantum wire systems with complex cross sections such as hemi-equilateral triangle (30–60–90 triangle), regular rhombus (supplementary angles of 60 and 120) and regular hexagon [36, 37]. The case of ETQD and ETQW with a finite barrier such as the ETQD and ETQW surrounded by other polar semiconductors is included in our future research project. It is well known that the ability to model the phonon modes in dimensionally confined structures has been the basis for efforts to design nanostructures such that the resulting carrier and phonon states are tailored to yield dissipative and scattering mechanisms different from those of the corresponding bulk structures [38]. We hope this paper will stimulate more theoretical and experimental work, which could be helpful for the study of the influence of phonons on physical properties in quantum dot and quantum wire systems of complex shapes.

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